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Enhancing Mathematical Problem-Solving Through Visualisation: Evidence from Ghanaian Preservice Teachers

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Abstract: Strengthening preservice teachers' mathematical problem-solving is important for the quality of mathematics teaching, particularly where prior learning has emphasised procedural fluency. This qualitative case study explored how preservice teachers' problem-solving processes and experiences changed during and after a structured visualisation intervention in Ghanaian colleges of education. Thirty preservice teachers, purposively selected from three colleges because they had scored below the institutional pass mark in a previous mathematics examination, took part in a four-week intervention comprising explicit visualisation strategies, guided problem solving, and reflective practice. Data were collected through semi-structured interviews, classroom observations, and analysis of participants' problem-solving artefacts, and were analysed using thematic analysis. Before the intervention, participants relied mainly on procedural methods and had difficulty understanding word problems. During and after the intervention, they translated problems more deliberately, created schematic visual representations, and showed more conceptual reasoning and strategic flexibility. No standardised measure of problem-solving performance was used, so the findings concern processes and experiences, not performance. On the basis of these findings, the study proposes an extension of Mudaly's Iterative Visualisation Thinking Cycle that adds an explicit Entry Stage, comprising problem translation and problem integration. The findings primarily concern preservice teachers experiencing mathematical difficulties and should not be generalised to all Ghanaian preservice teachers. The study offers context-specific evidence for mathematics teacher education research and practical suggestions for visualisation-based instruction.

Keywords: conceptual understanding; mathematical problem-solving; preservice teacher education; schematic visual representation; visualisation

INTRODUCTION

Mathematical problem-solving is a hallmark of mathematics learning, which involves mathematical reasoning, sense making and using mathematical knowledge in novel contexts (Schoenfeld, 2016; Lester & Cai, 2016). Despite these critical aspects, many students, even prospective teachers, struggle to be able to engage productively with non-routine problems, often becoming reliant on memorised procedures instead of concepts (Kilpatrick et al., 2001; Star, 2005). This is especially important for teacher education, during which preservice teachers' experiences in mathematics impact their own pedagogy and, in turn, the opportunities students have to acquire mathematical thinking (Ball et al., 2008; Hill et al., 2008).

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Recent curriculum changes in Ghana focus on pedagogies of learner-centred and inquiry-based learning, with an emphasis on understanding, reasoning and application (Ministry of Education, 2019; National Council for Curriculum and Assessment [NaCCA], 2019). The challenge of putting these policy goals into practice lies in the implementation in the classroom and the need to provide preservice teachers with specific, concrete strategies to facilitate conceptual understanding. Visualisation, as a pedagogical tool using internal mental imagery and external visual representations as support for reasoning, has been identified as a promising pedagogical tool for enhancing problem-solving and conceptual understanding (Arcavi, 2003; Presmeg, 2006). Despite the growing interest in visualisation in recent decades, however, reviews of this literature indicate that it is geographically concentrated and has focused mainly on school learners. A bibliometric review of 279 journal articles published between 2014 and 2024 reported that research output was concentrated in a small number of countries, with limited representation of low- and middle-income settings (Hadi et al., 2026). Work on preservice teachers' use of visualisation does exist in Sub-Saharan Africa, notably in South Africa (Zulu & Mudaly, 2023), but it is limited. To our knowledge, little is known about how preservice teachers use and adapt diagram-based visualisation strategies when they are taught explicitly for problem solving in Ghana.

The research in this study aimed to fill these gaps by looking at the experience of the preservice teachers in Ghana in solving problems, the adaptation of preservice teachers to visualisation strategies through structured learning, and how their problem-solving processes and experiences of problem solving changed during and after structured visualisation instruction. Theoretically, the study also contributes by further developing an existing visualisation framework to incorporate the preparatory cognitive process that supports successful problem-solving.

Research Questions

The following research questions were used to guide the study:

RQ1: What challenges do preservice teachers have in solving mathematical problems?

RQ2: What are the adaptations that preservice teachers make to the use of visualisation in mathematical problem-solving?

RQ3: How, if at all, do preservice teachers' problem-solving processes, and their experiences of problem solving, change during and after structured visualisation instruction?

LITERATURE REVIEW

In this paper, the research is synthesised on mathematical problem-solving, visualisation and preservice teacher education to provide a coherent conceptual foundation for the study. Instead of an

extensive list, this review highlights commonalities between these strands as a way of clarifying pre-service teachers' problems with problem-solving and the role of visualisation as a mediating strategy.

Mathematical Problem-Solving and Teacher Education

Mathematical problem solving is typically seen as work with problems where there are no obvious solution routes to take, involving interpreting problems, choosing a strategy, and reflecting on solutions (Pólya, 1945; Schoenfeld, 2016). One of the key factors in this process is the relationship between procedural knowledge (knowledge of rules and algorithms) and conceptual knowledge (knowledge of underlying principles and relationships) (Hiebert & Lefevre, 1986). While both types of knowledge are essential, research has consistently demonstrated that when it comes to non-routine problems, it is the focus on procedures that hampers learners' flexibility in reasoning (Lithner, 2008; Star & Stylianides, 2013).

This imbalance is significant, especially in teacher education. Preservice teachers' mathematical knowledge impacts their own problem solving and the problem-solving experiences that they offer students in their classrooms in the future (Ball et al., 2008). Research has shown that preservice teachers who are mostly procedural tend to have problems understanding students' reasoning, addressing their errors, or explaining their own solution methods conceptually (Ma, 1999; Thanheiser, 2009). These results indicate that enhancing preservice teachers' conceptual understanding of problems is key to teacher preparation.

Visualisation as a Mediating Tool in Mathematical Problem-Solving

Visualisation is well known as a facilitator of mathematical thinking since it allows learners to encode abstract relationships and to relate various mathematical activities (Arcavi, 2003; Duval, 2006). Visualisation is not an auxiliary aid, but rather a process of cognition in which meaning is built up that includes both mental images and external representations like diagrams, graphs, and models (Presmeg, 2006).

Theoretically, the use of visuals in problem solving can be explained by using two theories: Dual Coding theory (Paivio, 1986) and cognitive load theory (Sweller et al., 2011). Empirical studies in various mathematics domains have shown that effective visualisation is beneficial for understanding, problem interpretation, and conceptual understanding (Booth & Koedinger, 2012; Fyfe et al., 2014; Xie et al., 2020). The present body of work, however, also highlights the need to be careful with visualisations, since their value and utility rely on the way they are 'constructed', 'interpreted', and 'involved' in reasoning processes.

The concept of visualisation in Preservice Teacher Education

Although there is an increased awareness of its importance, research indicates that many preservice teachers lack experience in applying visualisation strategically in their learning and teaching (Stylianou, 2010; Avcu & Avcu, 2015). Preservice teachers' prior procedural learning experiences are often treated like a decoration or illustration. Without explicit learning of visualisation, preservice teachers may be unable to use it effectively for solving problems.

Recent research supports the necessity for explicitly teaching visualisation in teacher preparation programs, both in terms of preservice teachers' own problem-solving processes and in terms of their pedagogical understanding of scaffolding visualisation for learners (Göktepe-Yildiz & Özdemir, 2020). Reviews of visualisation research report that intervention studies are mostly short-term and that publication is concentrated in a limited set of countries (Hadi et al., 2026). This gives little insight into how preservice teachers in contexts such as Ghana adapt to visualisation strategies over time and how their problem-solving processes change.

Conceptual Positioning and Research Gap

Over the years, many studies have found that preservice teachers tend to approach mathematical problems in very procedural ways, focusing on how to do the algorithm and leaving the mathematical structures and relationships behind (Hiebert & Lefevre, 1986; Ma, 1999; Lithner, 2008; Star & Stylianides, 2013). On the other hand, visualisation is recognized as an important mediating process between the conceptual and external representations to facilitate the representation, interpretation and coordination of mathematical relationships (Arcavi, 2003; Duval, 2006; Stylianou, 2010; Fyfe et al., 2014).

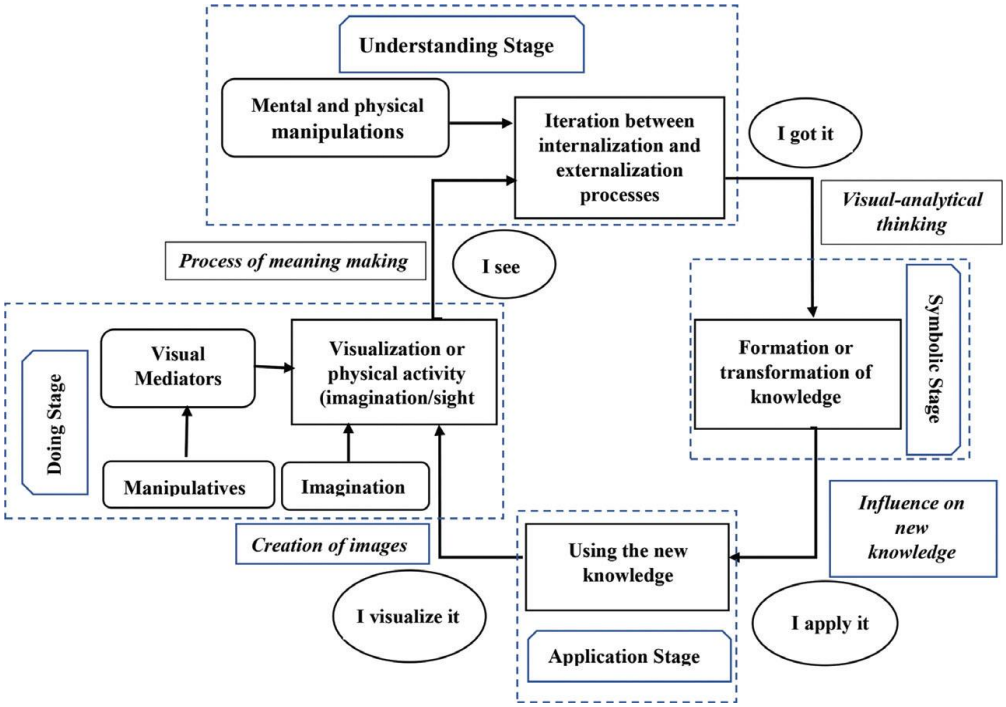
However, there are two gaps in the literature. First, existing evidence from Ghanaian preservice teachers concerns their perceptions of problem solving (Gyampoh, 2025) and GeoGebra-based courses (Agyei & Benning, 2014; Narh-Kert & Sabtiwu, 2022); qualitative evidence appears to be scarce on how preservice teachers experience and adapt diagram-based visualisation strategies that are taught explicitly for problem solving. Second, some frameworks describe the process of visualisation in an iterative way, but they do not go deep enough into the preparatory cognitive work required to conceptualise verbal problem statements as meaningful visual representations. This study addresses the first gap directly by investigating the challenges of problem solving, the strategies used to adapt to visualisation, and the changes in the problem-solving processes and experiences of preservice teachers. It then considers, in the discussion, whether its findings suggest a way of addressing the second gap by extending Mudaly's (2021) Iterative Visualisation Thinking Cycle.

Theoretical Framework

This study is based on the Iterative Visualisation Thinking Cycle developed by Mudaly (2021), which proposes four stages of learning through visualisation: Doing, Understanding, Symbolic, and Application. The framework represents the ongoing process of making meaning of visualisations. In this cycle, the four stages operate iteratively as learners move between visual or physical activity, internalisation and meaning-making, symbolic transformation, and application of new knowledge, which in turn feeds further visualisation. This framework served as the analytic lens for the present study; the Discussion returns to it in light of the empirical findings, presented below.

Figure 1. *Mudaly's Iterative Visualisation Thinking Cycle*

Note. Reproduced from "Constructing Mental Diagrams During Problem-Solving in Mathematics," by V. Mudaly (2021), Pythagoras, 42(1), Article a633 (https://doi.org/10.4102/pythagoras.v42i1.633), licensed under Creative Commons Attribution 4.0 International (CC BY 4.0).



METHOD

Research Design

This study used a qualitative case study approach (Yin, 2018), which aimed to explore the experiences of preservice teachers in authentic instructional settings with regard to the phenomenon of visualisation. This method allowed for a detailed analysis of cognitive and pedagogical processes, while taking into account context and meaning rather than measures of causal effects. The case is defined as

a single, bounded case: one four-week structured visualisation intervention, implemented in the same form and analysed together across three Ghanaian colleges of education, rather than as three separate institutional cases or as the thirty participants considered independently of one another. Consistent with an embedded single-case design (Yin, 2018), the three colleges are treated as embedded units of analysis within this one case, because the intervention, the data-collection procedures and the analytic approach were common to all three sites and the resulting data were analysed together rather than compared site by site. The boundaries of the case were as follows: place, three colleges of education in Ghana; participants, the thirty pre-service teachers purposively selected based on prior mathematics performance and availability (see participants and context, below); intervention, the eight-session, four-week structured visualisation programme described in the intervention section; and time, the four-week intervention period together with the pre- and post-intervention data-collection windows.

Participants and Context

Thirty students were purposefully selected from three Ghanaian colleges of education, based on two factors: namely, the results of their previous end-of-semester mathematics examination and their availability during the entire study period. Participants who had scored below 50%, the institutional pass threshold, were purposively selected (University of Cape Coast, n.d.). Purposive sampling (Patton, 2015) was used to ensure that participants included a variety of challenges to mathematical problem solving so as to maximise relevance of data to the research questions. Ethical approval was obtained before data collection from the Humanities and Social Sciences Research Ethics Committee (HSSREC) with approval number HSSREC/00003879/2022. All participants gave written informed consent after being told the purpose of the study, that participation was voluntary, that they could withdraw at any time without consequence, and that their participation and responses would be kept strictly confidential. Participant codes were used throughout data collection, analysis, and reporting to ensure confidentiality (that is, P01, P02, P03, P04, P05, …, P30). Because of this sampling criterion, the sample is a specific group of relatively low-attaining preservice teachers rather than a cross-section of the preservice teacher population. The findings reported below therefore primarily concern preservice teachers experiencing mathematical difficulties and should not be generalised to all Ghanaian preservice teachers; preservice teachers who perform well in mathematics may approach and use visualisation differently, and this possibility was not examined in the present study.

Intervention

The intervention was four weeks in duration and included eight 90-minute sessions that were woven into the regular coursework. The skills addressed in the instructional activities were tightly

interwoven, namely: (1) interpreting word problems by paraphrasing and identifying key relationships in the word problem; (2) constructing schematic diagrams and other visual mediators; (3) employing visual representation to plan and verify solution strategies; and (4) reflecting on the role of visualisation in supporting mathematical understanding and pedagogy. The problems were of an algebraic, geometric, and contextualised word-problem type and were both routine and non-routine. Researchers explicitly demonstrated visualisation strategies and supported participants' practice with graduated assistance during the intervention. The sequence of sessions is summarised in Table 1.

Table 1. Overview of the eight intervention sessions

Session	Mathematical topic	Visualisation strategy	Main activity	Reflection task
1	Geometry and Algebraic equations	Paraphrasing and annotating the problem	Re-reading, restating, underlining, or highlighting key information.	What did you do first when you read a problem?
2	Geometry and Algebraic equations	Mental imagery	Guide participants to picture the task in their head.	Think of an illustration that will depict the task given.
3	Geometry and Algebraic equations	Physical manipulatives	Concrete objects or paper folding.	What kind of physical manipulative can assist in solving the task?
4	Geometry and Algebraic equations	Sketching the problem situation (Schematics or structural diagrams)	Drawing the described scene or figure, such as the rectangle and diagonal in the worksheets.	Which schematic or structural drawing will illustrate the task given?
5	Geometry and Algebraic equations	Labelling and marking diagrams	Adding given values, the unknown, right-angle marks, and equal-side marks.	Compare the diagram to the task, ensuring it best describes the task.
6	Geometry and Algebraic equations	Extracting a sub-diagram	Pulling out the relevant structure, such as the right-angled triangle from the rectangle.	How can the task be simplified for easy understanding?
7	Geometry and Algebraic equations	Using visual representations to plan and verify solution strategies.	Evaluate the solution through verification of the visual representations.	Is the strategy effective for the task at hand?
8	Geometry and Algebraic equations	Reflecting on the role of visualisation	Comparing different diagrams of the same problem and discussing when each helps.	How did knowledge of visualisation help solve problems?

Data Collection

A rich, triangulated description of participants' experience was created using three data sources. Semi-structured interviews were undertaken with each participant prior to the intervention and following the intervention (around 30-40 minutes per interview), primarily around how participants perceived the problem-solving, use of representations, and experiences of the intervention. Second,

classroom observations took place during the sessions, and field notes were taken by the researcher of what verbal and written strategies participants used to solve problems, how they interacted with one another, and how they used visual representations. Third, artefacts such as worksheets, diagrams and written answers were gathered throughout the intervention and analysed alongside the interview and observational data.

Researcher Positionality and Reflexivity

Intervention, including pre-intervention and post-intervention interviews, and classroom observation, were carried out by researchers who were not lecturers at the colleges. Participants were given assurances of confidentiality and told that their responses would not affect their assessment at their colleges.

Data Analysis

Thematic analysis techniques outlined by Braun and Clarke (2006, 2019) were used to analyse the data. The analysis followed a reflexive thematic analysis approach, in which coding is treated as an interpretive and iterative process rather than one aimed at establishing measurable coder reliability; coding was both inductive (allowing themes to emerge from the data) and deductive (informed by the research questions and by Mudaly's (2021) framework). The first author initially coded all interview transcripts, observation field notes, and artefacts. To support the credibility of the analysis, the second author independently coded 60% of the material, selected at random, before we compared their codes. The second author's coding was carried out without seeing the first author's codes. Discrepancies in coding were discussed by the two authors in a couple of meetings until consensus was reached by returning to the original data extract and checking it against the research questions. Because the study followed a reflexive thematic analysis approach, intercoder agreement was not calculated as a reliability statistic; instead, the second author's independent coding and subsequent discussion served as an analytic check on the first author's interpretations, consistent with Braun and Clarke's (2019) account of this approach. Final themes and subthemes were agreed by both authors through a final round of comparison against the research questions and the raw data, and were developed through constant comparison across data sources, moving back and forth between the data, codes, and emerging themes until a coherent and grounded account was reached. Member checking (Lincoln & Guba, 1985) was carried out with twelve participants; four participants were selected from each college.

These participants were sent a short written summary of the preliminary themes and were asked to confirm whether the summary reflected their experience and note anything that felt inaccurate or incomplete. The participants confirmed that the transcripts were accurate and a true reflection of what transpired during the interview (responses provided by participants). Table 2 summarises some of the

themes and subthemes generated through this process, their correspondence to the research questions, and examples of supporting evidence.

Table 2. Themes, subthemes and supporting evidence

Theme	Subtheme	RQ	Example evidence	Data source
Procedural orientation to problem solving	Solving problems using repeated procedures.	RQ1	P01: "I master the problem by solving the problem repeatedly, and this helps make solving problems much easier for me".	Interview
	Relying on formulas to solve problems.	RQ1	P02: "I use formulas and rules to solve problems".	Interview
Difficulty interpreting word problems	Difficulty identifying the mathematical structure within the problem narrative	RQ1	P11: "Sometimes, it takes me some time to understand the mathematics concepts in word problems, and this makes problem-solving difficult for me".	Interview; Observation
Limited use of visual representation	Visuals, when produced, inaccurate, incomplete, or not used as a tool for reasoning	RQ1	See worksheet of P11 and P05	Artefact
Problem translation	Re-reading and paraphrasing the problem statement as a preliminary step	RQ2	P15: "Read the problem more than once to really understand it. Put the problem into your own language".	Interview; Observation
Problem integration	Constructing a schematic diagram to externalise and clarify relationships	RQ2	P06: "I break down the question to make it simpler and easier to understand. After that, I draw diagrams that represent the problem accurately".	Interview; Artefact
	Using the diagram to plan and check a solution strategy	RQ2	P08: "I start by turning the question into a diagram, and then I look for different methods to solve the problem".	Observation; Artefact
Perceived ease of processing	Visual representations described as making problems easier to work with.	RQ3	P11: "After going through the process of visualisation, I can picture problems in my mind, and that makes it easier to understand and solve them".	Interview
Increased conceptual explanation and self-monitoring	More conceptual explanation of solutions and visible self-checking during sessions	RQ3	It was observed that using visualisation improves conceptual understanding, and participants were able to reflect on their own work.	Observation

RESULTS AND DISCUSSION

Findings are presented below by research question. Each major theme is illustrated with direct evidence from at least two of the three data sources (semi-structured interviews, classroom observations and problem-solving artefacts).

Difficulties in Mathematical Problem-Solving (RQ1)

Before the intervention, participants mainly took a procedural approach when solving problems and had an understanding of mathematics through repeatedly performing the problem. The following are extracts from the interview.

P01: *"I master the problem by solving the problem repeatedly, and this helps make solving problems much easier for me".*

P06: *"I get better at solving a problem by doing it over and over again because the more I practice, the easier it gets for me".*

P11: *"If I keep solving the same types of questions over and over, I become good at the formulae needed to solve them".*

Repeatedly solving problems can be a useful learning technique, but it is not without some drawbacks. Over-reliance on repetitive approaches to problem-solving might result in specific learning challenges and restrictions. According to Bransford et al. (2000), solving problems repeatedly can improve fluency and retention of particular problem-solving approaches, but it may not always make it easier to apply what has been learned to new situations or problem types. Particularly if they have only come across a limited variety of problem types in their practice, learners may find it difficult to generalise their problem-solving techniques and apply them to novel situations.

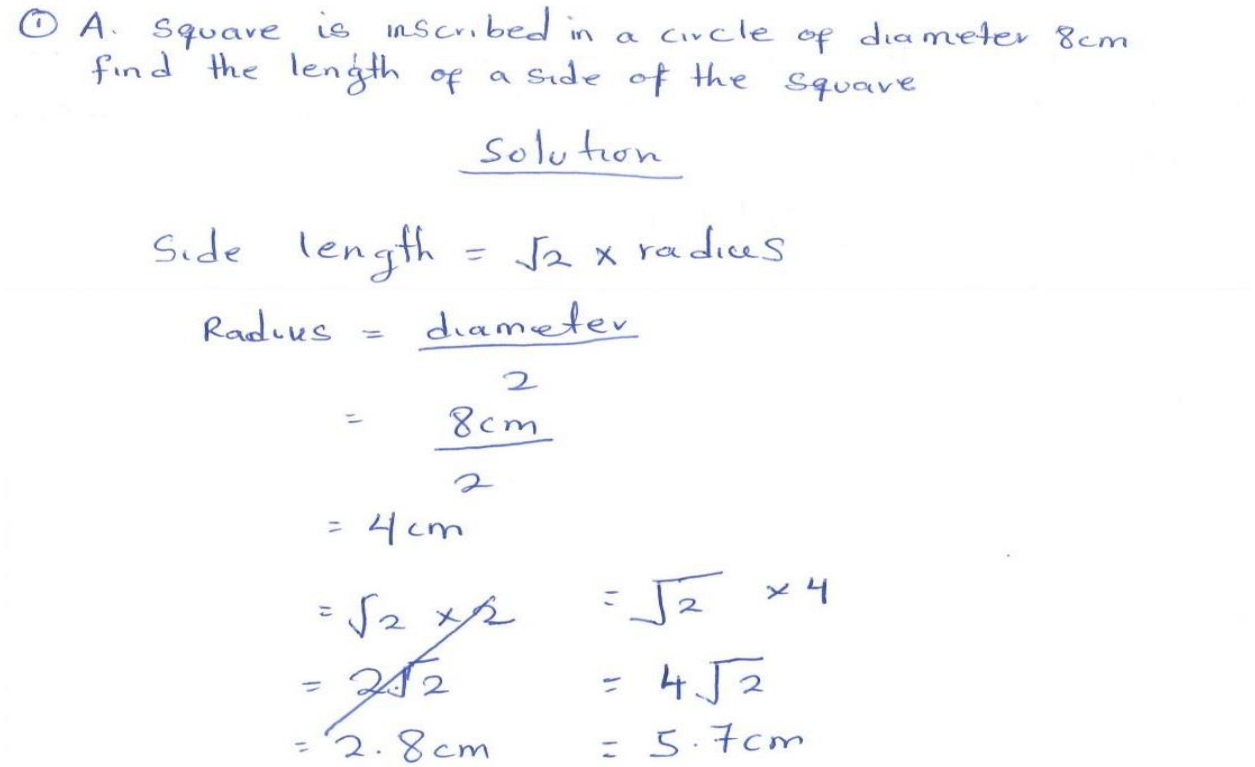
Many participants had considerable difficulty understanding word problems, especially in identifying the mathematical structures within the problem narratives. The following are extracts from participants.

P05: *"I often face challenges in following step-by-step procedures in mathematics or understanding concepts in word problems, and that makes problem-solving difficult for me".*

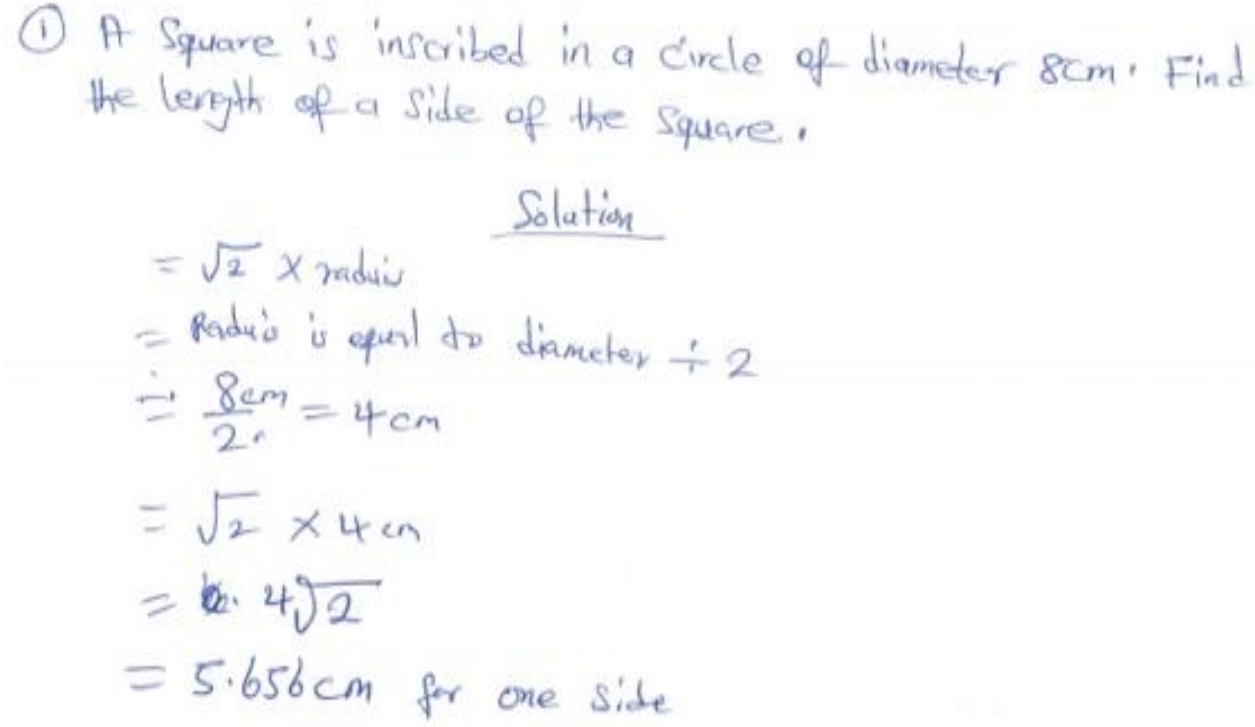
P11: *"Sometimes, it takes me some time to understand the mathematics concepts in word problems, and this makes problem-solving difficult for me".*

In addition, during classroom observation, it was inferred that participants had challenges in deducing mathematical formulation from mathematical word problems. These observations were evident from participants' worksheets.

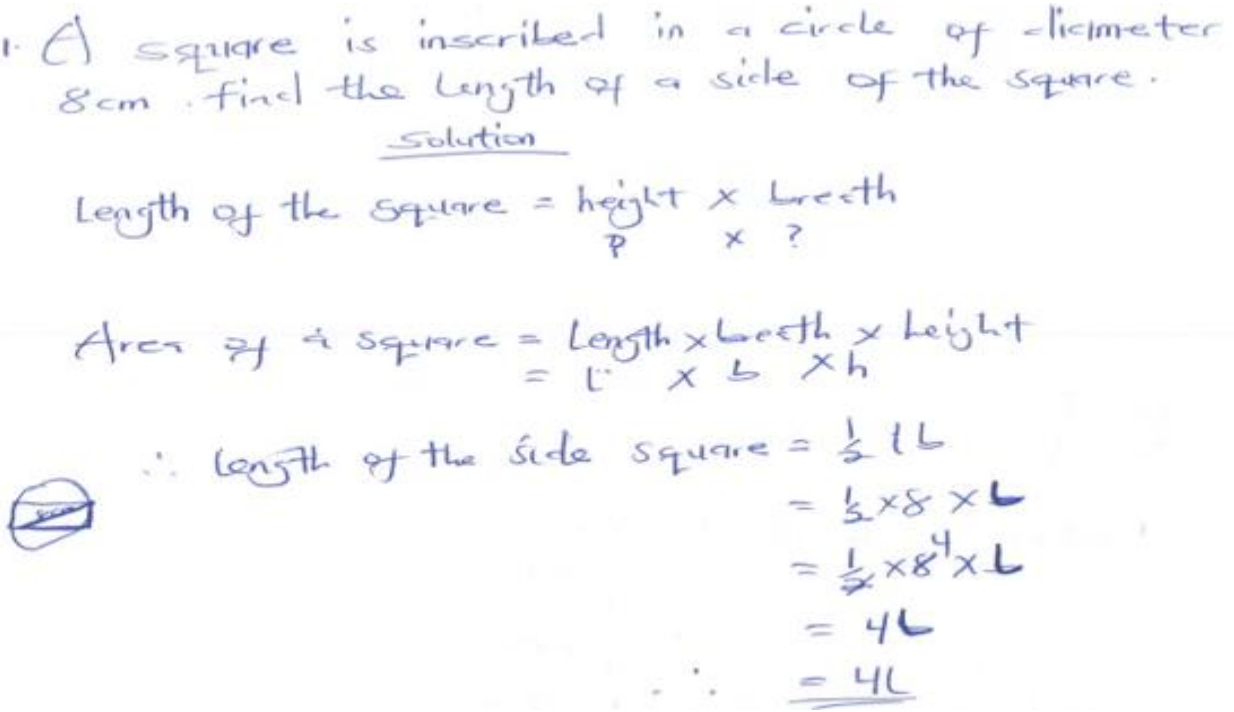
Visual representations were not often used spontaneously and, when used, they were rarely accurate, complete, or treated as a tool for reasoning. Below are samples of students' work before the intervention.



Worksheet from P11



Worksheet from P05



Worksheet from P06

The findings are in line with previous studies that reported preservice teachers' procedural orientations as well as their restricted use of visual representations in problem-solving (Ma, 1999; Stylianou, 2010; Avcu & Avcu, 2015).

Students' adaptation to Visualisation Strategies (RQ2)

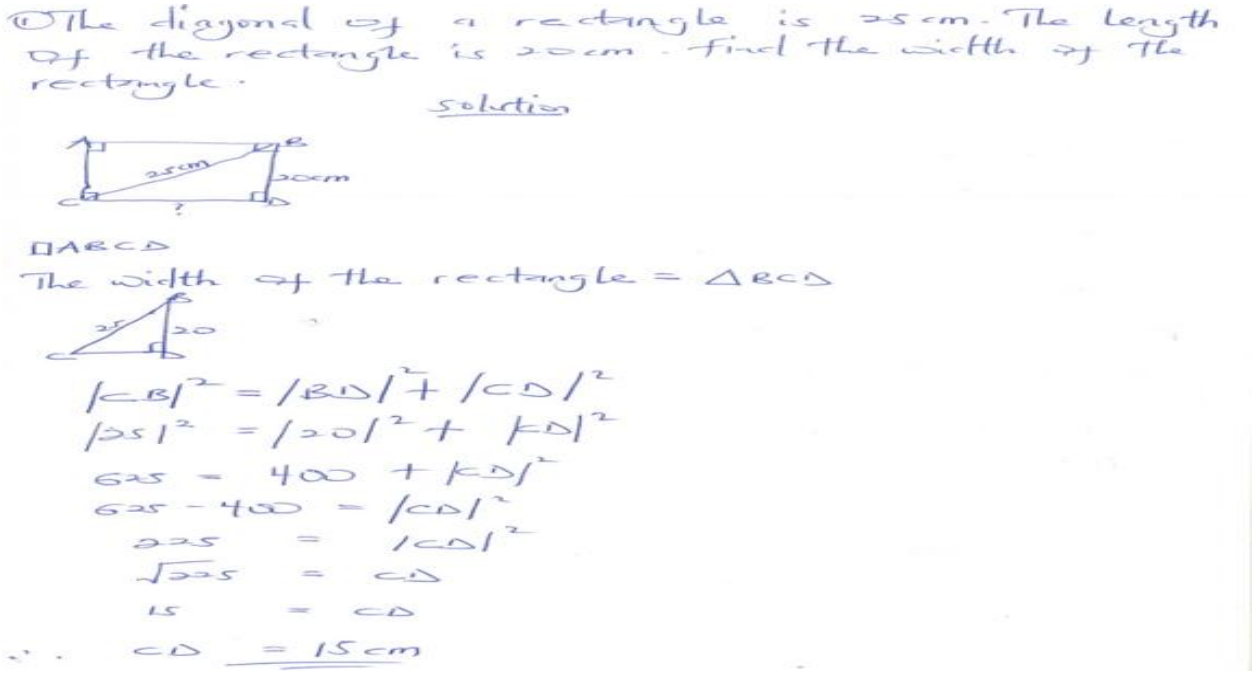
Visualisation techniques were introduced progressively, with guidance, leading to more conscious and intentional use. During the intervention, participants mentioned that they re-read and paraphrased problems as a preliminary step, built schematic diagrams to externalise and clarify the relations, and used these to inform the planning of solution strategies. Below are excerpts from participants.

P11, *"First, I read the problem several times to really get what it's asking". Then, I put it in simpler words to make sure I understand. After that, I create a mental image of what the problem might look like. Finally, I figure out a way to solve it based on what I have visualised.*

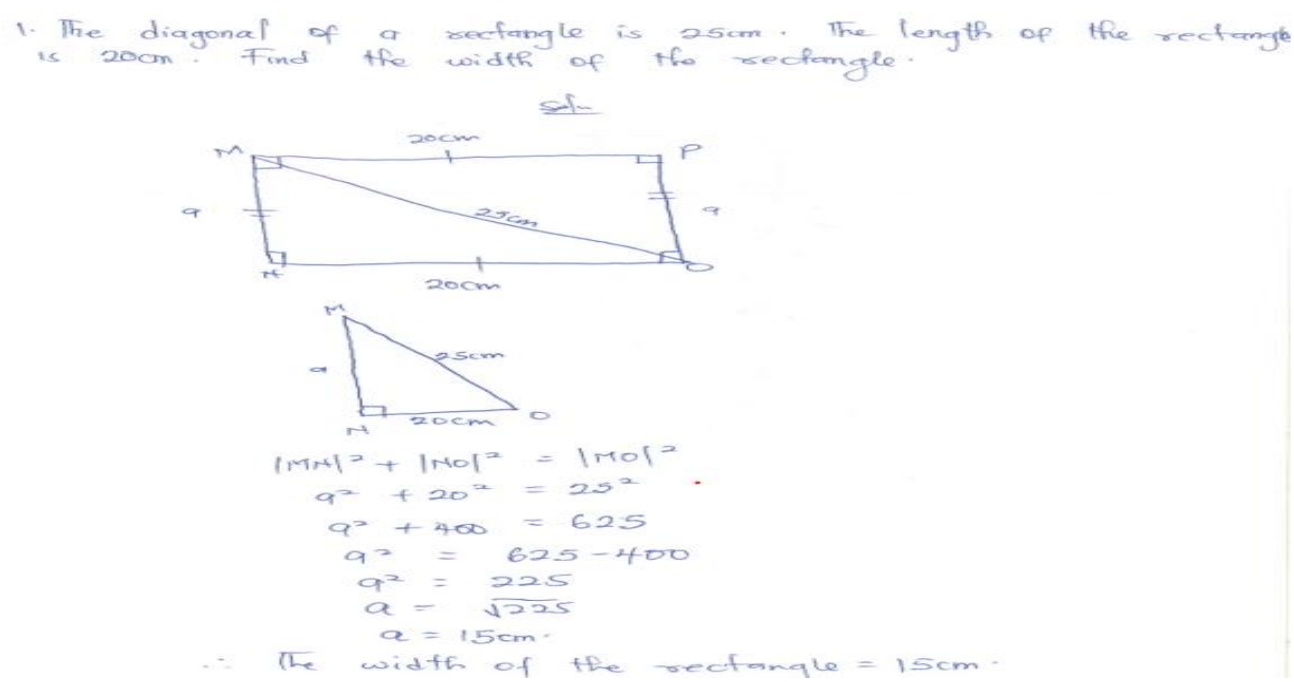
P13: *"I take my time to understand the problem by reading it more than once. Make a simple drawing or diagram to help me see the problem visually. Consider different ways to solve the problem and choose the best way based on what you learn from the problem and your drawing".*

P14: *"I read the problem more than once and underline or highlight the important parts that stand out to you. Picture the elements and relationships in your head and draw a simple sketch or diagram to help you work through the problem".*

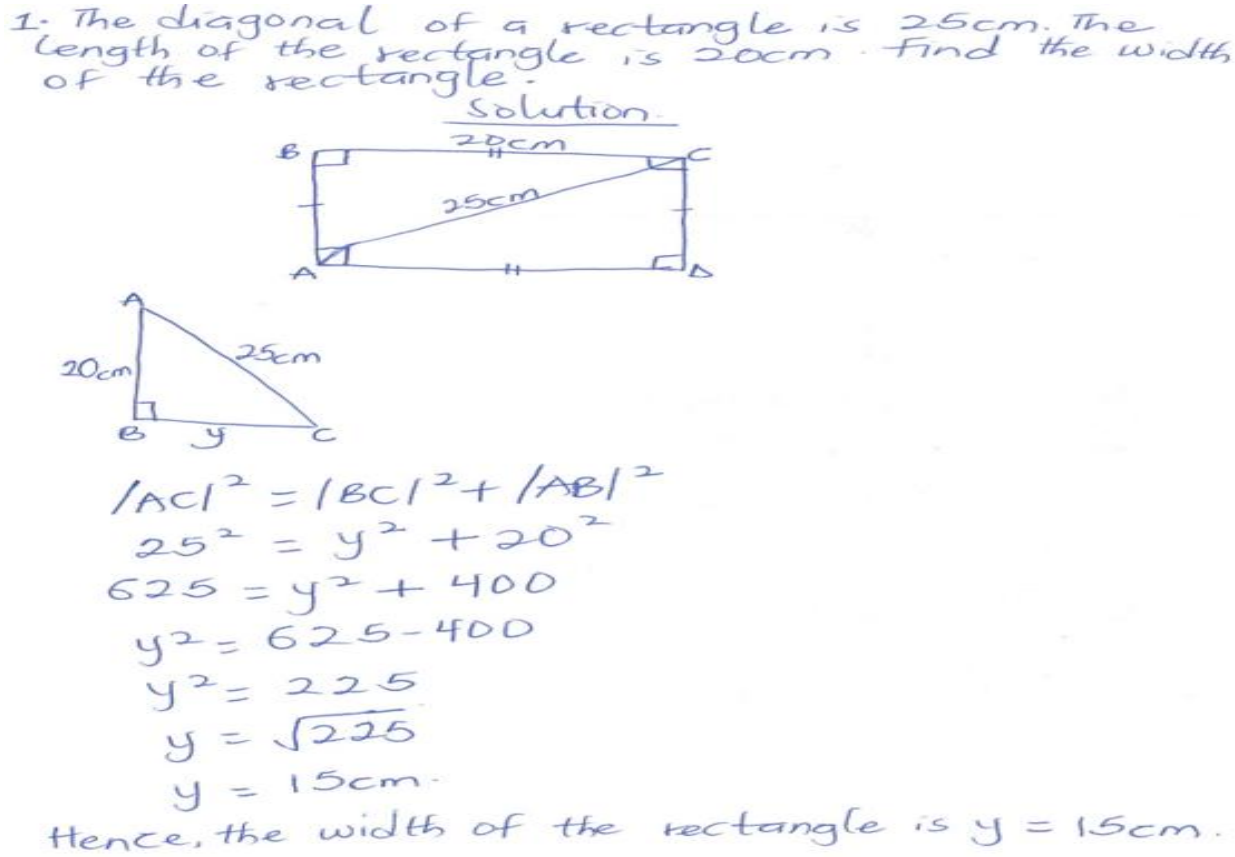
An external system of visual mediators was favoured over internal visuals, especially with complex and multi-step problems. This shift from the incidental to systematic use of visualisation matches the learning pathways reported by Göktepe-Yildiz and Özdemir (2020) and the learning paths outlined in Mudaly's (2021) framework. Below are samples of students' work after the intervention, where they made effective use of visual mediators.



Worksheet from P05



Worksheet from P06



Worksheet from P11

Analytic Commentary on the Worksheets

To make explicit what changed in each case, the pre- and post-intervention worksheets are analysed artefact by artefact below.

P05. *Before:* P05 recalled the relation side = √2 × radius and correctly computed 4√2 ≈ 5.656 cm but drew no diagram and did not construct or state why the side of the square relates to the radius by this factor; the underlying relationship between the diagonal of the square and the diameter of the circle, which would justify the formula, is never represented.

After: P05 drew the rectangle together with its diagonal and known sides, separated the right-angled triangle formed by the diagonal, labelled the unknown side, and applied the Pythagorean theorem to it, carrying the representation through to the numerical solution (CD = 15 cm) and stating it in the context of the problem.

P06. *Before:* P06 attempted to work from an area-based formula (length × breadth) without constructing any diagram; the work became incoherent and ended in an unresolved expression (4L), with no numerical answer and no geometric justification for the steps taken.

After: P06 drew the rectangle, marked the equal opposite sides and right angles, separated the right-angled triangle formed by the diagonal, and applied the Pythagorean theorem to reach the correct numerical answer (15 cm), again stating the result in the context of the problem.

P11. *Before:* P11 also presented the work similar to P05. The participant used the relation $\sqrt{2} \times$ radius and correctly computed $4\sqrt{2} \approx 5.656$ cm but drew no diagram and did not construct or state why the side of the square relates to the radius by this factor; the underlying relationship between the diagonal of the square and the diameter of the circle, which would justify the formula, is never represented.
After: P11 drew the rectangle with its diagonal, marked the equal sides and right angles, separated the right-angled triangle, applied the Pythagorean theorem directly to the triangle, and stated the final answer explicitly in the context of the problem ("the width of the rectangle is y = 15 cm").

Across all three participants, the shift is from adapting a formula with no supporting diagram to constructing a diagram, separating out the relevant right-angled triangle, and explicitly connecting that representation to the numerical solution.

The handwritten examples above are drawn from the same three participants throughout (P05, P06 and P11), so that each participant's pre- and post-intervention work can be compared directly. These participants were selected because their work was representative of the wider sample of thirty participants. The pre- and post-intervention tasks were not equivalent, and no standardized measure of improvement was administered. Because the pre- and post-intervention tasks differ, these worksheets illustrate the kinds of approaches observed before and after the intervention rather than providing a controlled comparison of the same task solved twice.

Changes in Problem-Solving Processes and Experiences (RQ3)

Several participants reported that visual representations made problems easier to process and work with, and that visualisation helped them understand the problem situation and check their solutions. Participants made these submissions in response to the interview question.
P01: "*Thinking in pictures has made mathematics problem-solving easier because it helps me understand and solve problems better*".
P11: "*After going through the process of visualisation, I can picture problems in my mind, and that makes it easier to understand and solve them*".
P05: "*After using visualisation, I can now make difficult problems simpler and easier to understand*".

P06: *"Solving problems has become easier for me because now I can picture them in my mind, and that really helps".*

These reports were consistent with the observations, which showed more conceptual explanation in problem solving and self-monitoring behaviour, as well as flexibility in selecting and adapting solution strategies. The consistency across the interview data, the observational evidence, and the artefact analysis suggests that these preservice teachers' problem-solving processes changed during and after the intervention. They interpreted problems more deliberately, represented relationships more explicitly, and selected and adapted strategies more flexibly (Booth & Koedinger, 2012; Fyfe et al., 2014).

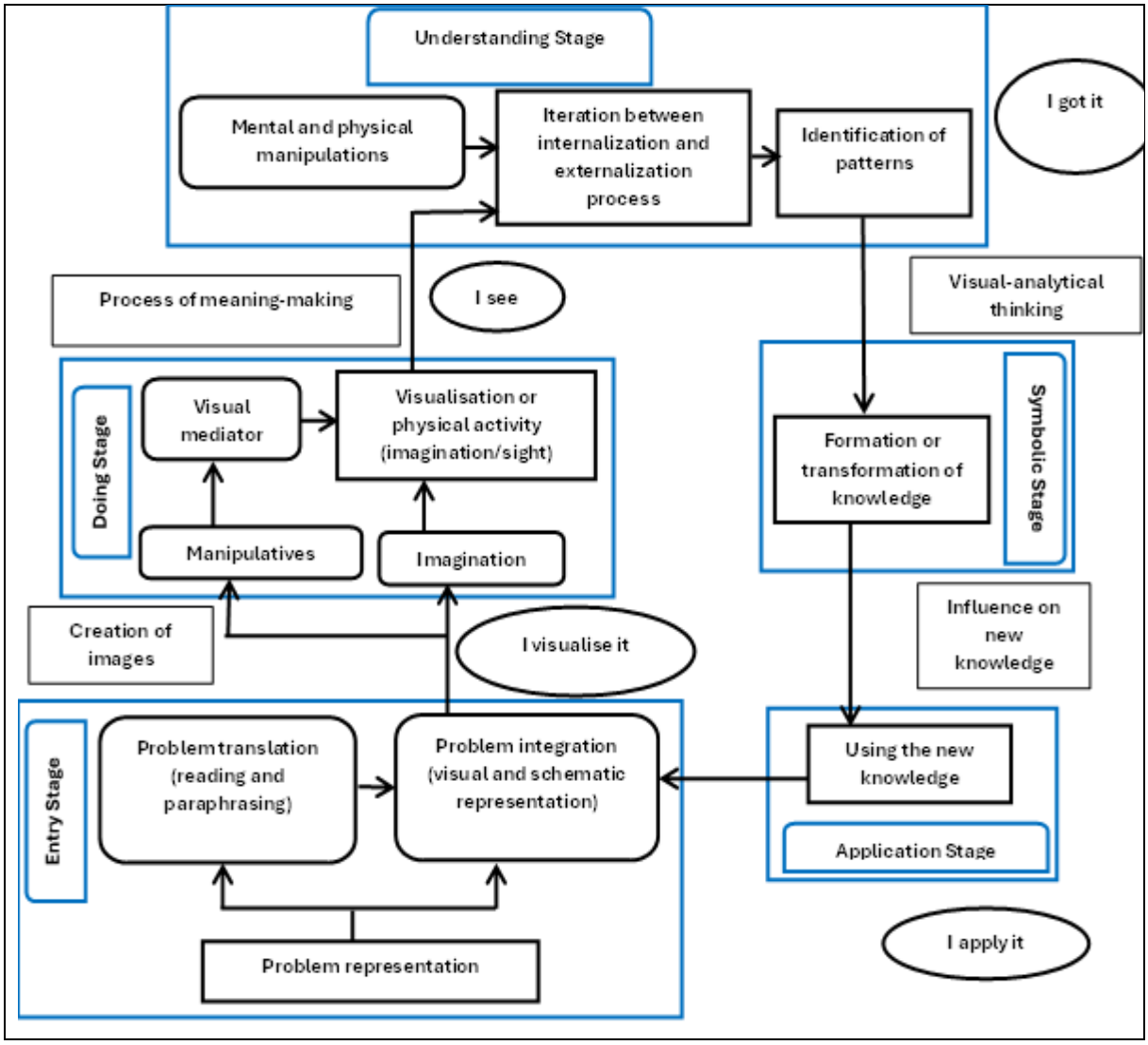
Theoretical Contribution: An enhanced visualisation framework

The Theoretical Framework (Figure 1) described Mudaly's (2021) four-stage cycle, beginning with the Doing Stage, in which learners already work with a visual mediator. The findings reported above indicate that, for these preservice teachers, something needed to happen before that stage. Participants who struggled to interpret word problems before the intervention (RQ1) rarely produced a visual representation at all. Following the intervention (RQ2), participants who did visualise successfully described first re-reading and paraphrasing the problem (e.g., P11, P13 and P14, above) and then building an initial schematic representation of its relationships before using that representation to plan a solution strategy, a sequence also visible in their worksheets. This preparatory work is not accounted for by Mudaly's (2021) cycle, which begins once a visual mediator is already in use. On this basis, we propose refining the Iterative Visualisation Thinking Cycle to make this preparatory work explicit. The proposed enhancement adds a new Entry Stage composed of two sub-processes identified in the data: Problem Translation (paraphrasing and clarifying the problem situation) and Problem Integration (building an initial schematic representation that reveals the relational structure and constraints of the problem). This stage represents preparatory work that is specific to visualisation, and that appears, in these findings, to influence the quality of the visual reasoning that follows. Table 3 summarises this evidence by component, showing that each draw on multiple participants and more than one data source.

Table 3. Evidence for the proposed Entry Stage from the qualitative data

Component	Participant(s)	Data source	Illustrative evidence
Problem Translation	P11	Interview	"First, I read the problem several times to really get what it's asking. Then, I put it in simpler words to make sure I understand."
	P13	Interview	"I take my time to understand the problem by reading it more than once."
	P14	Interview	"I read the problem more than once and underline or highlight the important parts that stand out to you."
	P16	Observation	Participant P16 was seen reading about the problem several times and underlined the important parts that stand out.
Problem Integration	P11	Interview	"...I create a mental image of what the problem might look like. Finally, I figure out a way to solve it based on what I have visualised."
	P13	Interview	"Make a simple drawing or diagram to help me see the problem visually and choose the best way based on what you learn from the problem and your drawing."
	P14	Interview	"Picture the elements and relationships in your head and draw a simple sketch or diagram to help you work through the problem."
	P05, P06, P11	Artefact	Post-intervention worksheets: each participant drew the rectangle with its diagonal and known sides, separated out the right-angled triangle, and used it to set up and solve the Pythagorean relation (see worksheets above).

Figure 2. Proposed Extended Visualisation Framework



The proposed Entry Stage is deliberately framed as a refinement within Pólya's (1945) first phase, understanding the problem, rather than as a separate stage of problem solving, and the distinction is stated precisely here rather than asserted. Pólya's first phase is a general heuristic. It directs the problem solver to identify the unknown, the data and the condition, and it suggests restating the problem in one's own words, drawing a figure and introducing suitable notation as ways of doing this. It does not specify how a verbal problem statement is transformed into a schematic representation, nor does it separate the processes involved in doing so. The Entry Stage makes three such processes explicit. First, Problem Translation is a linguistic and conceptual process of re-reading and paraphrasing the problem statement into the problem solver's own words (Table 3: P11, P13, P14). Second, within this translation, the data point to a further process of identifying the relational structure of the problem. That is working out which quantities are related, and how before any representation is built. Third, Problem Integration

is a representational process, in Duval's (2006) sense of conversion between registers of representation: the transformation of this verbal and relational understanding into an external schematic representation (a diagram, labelled figure or sketch), and the coordination of that representation with the verbal statement so that the two can be checked against each other (for example, P13's description of choosing a solution method "based on what you learn from the problem and your drawing"). Pólya's suggestions of restating the problem and drawing a figure name two possible actions without specifying this translation – to – integration sequence, the register – conversion work it involves, or why, for the preservice teachers in this study, failure at this point prevented visualisation from beginning at all (RQ1, above). The Entry Stage therefore does not claim to replace Pólya's phase; it specifies, for visualisation – based problem solving in particular, what "understanding the problem" requires in practice, and shows, through the data summarised in Table 3, that linguistic translation, identification of relational structure, and the transformation and coordination of verbal and visual representations are separable, observable processes rather than a single undifferentiated step. This refinement also addresses a gap in current visualisation models, which tend to assume that the structure of a problem is already accessible to the learner and give limited attention to the representational work required to make it so (Duval, 2006; Stylianou, 2010).

Limitations

Participants were purposively selected because they had scored below 50%, the institutional pass threshold, in a previous mathematics examination, and they came from three colleges of education. The findings therefore concern preservice teachers experiencing mathematical difficulties and cannot be generalised to all Ghanaian preservice teachers.

Implications

Implications for teacher preparation

Teacher education programs should explicitly and systematically teach visualisation, both cognitively, as a construct of visual reasoning, and pedagogically, as a tool to scaffold visualisation for various learners. This type of instruction should be part of the mathematics methodology courses and should be explicitly evaluated, not as an incidental part of mathematical competence. It is recommended that structured tasks that ask preservice teachers to engage with visual representations progressively and reflectively be utilized to develop their visualisation competencies.

Policy and contextual implications

The results are in line with Ghana's curriculum reform agenda (Ministry of Education, 2019; NaCCA, 2019), which seeks evidence for strategies that will help mathematics teachers implement learner-centred teaching and learning in mathematics. It implies that investment is needed in instructional

resources and in the professional development of teachers, as well as a coherent connection between initial teacher education and curriculum policy, to enable the effective and sustained application of visualisation-based pedagogy

CONCLUSION

With explicit and systematic teaching of visualisation, this study offers qualitative evidence that structured visualisation instruction was associated with noticeable changes in these preservice teachers' approach to mathematical problem-solving. Before the instruction, participants' answers were limited, and they used procedural repetition and limited engagement with problem contexts, which limited their ability to solve non-routine tasks. Preservice teachers' problem translation and representation skills through structured visualisation instruction were enriched in their conceptual reasoning and strategic flexibility.

The main theoretical contribution of the study is the introduction of an explicit Entry Stage to Mudaly's (2021) Iterative Visualisation Thinking Cycle, which emphasizes problem translation and integration. This refinement extends the existing models of visualisation by highlighting that the process of creating an effective visualisation is dependent on the initial interpretation and representation of the problem, not on the spontaneous process of problem-solving. Importantly, the Entry Stage is not intended to be general problem-solving heuristics, as it is proposed to be visualisation-specific.

The findings, from a practical perspective, reinforce the importance of teacher education programmes to go beyond assuming visualisation skills are something which develops intuitively among students. Pre-service teachers need specific and progressive experiences to develop their ability to make sense in picture terms of problem situations, and to analyze how to use these pictures with future learners. In a setting where curriculum is undergoing significant changes in Ghana, this study provides evidence-based support for curriculum alignment between learner-centred policy agendas and practice in the classroom. More generally, the study adds to the international debate on mathematics teacher preparation in the context of mathematics education in the Global South, where studies on visualisation are scarce (Hadi et al., 2026).

Additionally, mixed methods or longitudinal designs should be used to explore the sustainability of the visualisation competencies over the entire practicum and early career stages of teacher preparation, and to determine how the visualisation competencies are applied in classroom instruction. Generalising the findings would be strengthened by comparative studies across various contexts of teacher training in Ghana and other parts of the sub-Saharan region.

Conflict of Interest

The authors declare that they have no known competing interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available upon request.

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No funding was obtained for the work.

Ethics

The Humanities and Social Sciences Research Ethics Committee (HSSREC) approved the study before data collection (approval number HSSREC/00003879/2022).

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